Inflation and the Exchange Rate: The Role of Aggregate Demand Elasticity

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Publication Information  

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Abstract
This paper examines the effect of aggregate demand elasticity on the exchange rate when inflation occurs. We discover that both the source of the inflation, whether demand-pull or cost-push, and the elasticity of aggregate demand with respect to the price level, are of consequence for the exchange rate. We obtain two primary conclusions. First, the effect on the exchange rate of cost push inflation is ambiguous and is partially determined by the price level elasticity of aggregate demand. In particular, and assuming that the two examined countries have equivalent aggregate supply elasticities, we conclude that the nation with the less elastic aggregate demand function will see its currency appreciate relative to the other. Second, demand-pull inflation results in an unambiguous increase in the exchange rate but the size of that increase is partially a function of aggregate demand elasticity. Assuming again that two countries have equivalent aggregate supply elasticities, that country with the more elastic aggregate demand will experience currency appreciation.

Disciplines
Economics

Comments
The final publication is available at Springer via http://dx.doi.org/10.1007/s11294-015-9544-x
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Introduction

It is rather well established in macroeconomic theory that domestic inflation, ceteris paribus, relates to home currency depreciation. A rising price level in the short-run, however, may originate from either increased aggregate demand or decreased aggregate supply and these different causes of inflation have opposite effects on real income and in turn the exchange rate. A concept largely ignored in macroeconomics which may have some significance here is the price level elasticity of aggregate demand\(^1\) since the resultant changes in both the price level and real income for given shocks to aggregate demand and aggregate supply depend partially on this particular elasticity.

Previous examinations of the connection between inflation and the exchange rate have referred tangentially to the purchasing power parity theory of the exchange rate without consideration of the price level elasticity of aggregate demand. For example, Cassel [4] related international differences in inflation rates to differences in money supply growth rates between

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\(^1\) The literature on the price level elasticity of aggregate demand is indeed scarce. Gambs [8] has shown that the Classical school, from its emphasis on the quantity theory of money, implied that aggregate demand is unit elastic with respect to the price level. Keynes [10] and his early followers believed that aggregate demand was perfectly price level inelastic for the liquidity trap situation. Havrilesky [9] and Purvis [20] derived expressions for the price level elasticity of aggregate demand within the standard price-flexible IS-LM model. Using an expanded IS-LM framework to include the Pigou effect, Kyer and Maggs [13] also derived an expression for the price level elasticity of aggregate demand. Kyer and Maggs have additionally demonstrated the relevance of this elasticity for different monetary policy rules when aggregate supply shocks occur [14] and for the validity of supply-side economics [15]. More recently, Kyer and Maggs have shown the relevance of aggregate demand elasticity for the federal government budget deficit [17] and the balance of trade [18]. Empirical estimates of this particular elasticity are even more limited than the theoretical discussions. A few estimates (Klein[11]) which seem to suggest that aggregate demand in the United States is inelastic with respect to the general price level were derived by estimating the IS-LM cores of various large-scale macroeconomic models. Kyer and Maggs [16], however, have also estimated this elasticity for the United States and concluded that aggregate demand was price level elastic for most of the time period from 1955 to 1991. Apergis and Elestherio [1] estimated this elasticity for the Greek economy.
nations but did not address the possibility that the two countries with the same rates of nominal monetary expansion might experience different inflation rates because of differences in their respective price level elasticities of aggregate demand. We contend that the price level elasticity of aggregate demand is a relevant determinant of home and foreign price levels and real income and thus the exchange rate when increases of the price level are caused by either aggregate demand or aggregate supply shocks. This paper, therefore, explicitly studies the importance of aggregate demand elasticity for changes to international price levels and correspondingly the exchange rate when both aggregate demand and aggregate supply shocks occur.

The Model

This paper employs a conventional short-run aggregate demand, aggregate supply model of two economies, home and foreign, without governments.\(^2\) Aggregate demand for each country is assumed negatively dependent on the general price level but to demonstrate different price level elasticities. The aggregate supply of final goods and services is specified as a positive function of the price level, which implies some degree of wage or factor price rigidity. We assume further that the exchange rate is a positive function of the foreign price level and is inversely related to the domestic price level.

Let the home and foreign economies be described by the following pairs of reduced form aggregate demand-aggregate supply equations:

\[
\begin{align*}
y^d &= a_0M^{\alpha_d}P^{\alpha_d} \eta (\alpha^d) \\
y^f &= c_0M^{\alpha_f}P^{\alpha_f} \eta_f (\alpha^f)
\end{align*}
\]

\[
\begin{align*}
y^s &= b_0P^{\beta_s} \eta (\alpha^s) \\
y_f &= e_0P^{\beta_f} \eta_f (\alpha^f)
\end{align*}
\]

where \(y^d\) is the aggregate demand for goods and services and \(y^s\) is the aggregate supply of goods and services for the home country, and those variables with the “f” subscript represent the

\(^2\) The analysis which follows is in the spirit of other “Monetary Approach …” models, including Frenkel [6] and Bilson [2], along with those discussed in Boughton [3].
foreign country. The nominal money supply is M, P is the aggregate price level, and $\eta$ and $\gamma$ are exogenous shock variables for aggregate supply and aggregate demand, respectively, and $\alpha^e$ is a price-expectations parameter relating to supply behavior.\(^3\)

Linearizing the above static, non-stochastic system results in the following equations, where all variables are expressed in natural log form:

$$y^d = a_0 + a_1 M + a_2 P + \gamma \quad (3) \quad y^d_f = c_0 + c_1 M_f + c_2 P_f + \gamma_f \quad (3f)$$

$$y^s = b_0 + b_1 P + \eta(\alpha^e) \quad (4) \quad y^s_f = e_0 + e_1 P_f + \eta_f(\alpha^e) \quad (4f)$$

Standard macroeconomic theory suggests the inequality constraints of $a_0$, $a_1$, $b_0$, $c_0$, $c_1$ and $e_0 > 0$ and $a_2$ and $c_2 < 0$. In order to frame aggregate supply in the short-run, it is also assumed that $b_1$ and $e_1 > 0$, permitting us to focus primarily on the price level elasticity of aggregate demand and only secondarily on the price level elasticity of aggregate supply for price level and exchange rate determination. From these it follows that $a_1$ and $c_1$ are monetary elasticities of aggregate demand, or income velocities of money, in the home and foreign economies, respectively, while $a_2$ and $c_2$ are the price level elasticities of aggregate demand in the two countries. Similarly, $b_1$ and $e_1$ are the price level elasticities of aggregate supply in the home and foreign nations.

Solving equations (3), (3f), (4) and (4f) for the equilibrium levels of $y$, $P$, and $M$ in the two countries yields:

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\(^3\) The classic references on the role of price expectations for the behavior of aggregate supply are Friedman [7] and Lucas [19]. A recent approach to modelling aggregate supply focuses on potential output and the output gap and augments a standard production function with an inflation equation which relates inflation in the current period partially to long-run inflation expectations. For more detail on the “unobserved components model” of the economy’s supply side see Reifschneider, Wascher, and Wilcox [21], Fleischman and Roberts [5] and Kuttner [12].
As expected, the price level and real income in both countries vary positively with aggregate demand changes. Further, real income in the two countries also varies positively with changes of aggregate supply but the price level varies negatively with aggregate supply shocks. Real income and the price levels of both countries also vary positively with changes in the respective nominal money supplies.

The exchange rate, \( \pi \), is specified as the ratio of the foreign price level to the domestic price level, or

\[
\pi = \frac{P_f}{P} = \frac{e_0 - c_0 + (e_1 - c_2)P_f + \eta_f(\alpha^*) - \gamma_f}{c_1} \tag{8}
\]

It follows that a lower (higher) \( \pi \) would indicate relative inflation (deflation) in the home (foreign) nation, or home currency (foreign) depreciation (appreciation). From equation (8), the rate of change of the exchange rate is shown to equal the difference between the foreign and home country inflation rates.
Linearization of equation (8) gives

\[ \pi = P_f(M_f, \eta_f(\alpha^e), \gamma_f) - P(M, \eta(\alpha^e), \gamma) \quad (8') \]

and total differentiation of equation (8') yields:

\[ d\pi = \frac{\partial P_f}{\partial M_f} dM_f + \frac{\partial P_f}{\partial \eta_f} \frac{\partial \eta_f}{\partial \alpha^e} d\alpha^e + \frac{\partial P_f}{\partial \gamma_f} d\gamma_f - \frac{\partial P}{\partial M} dM - \frac{\partial P}{\partial \eta} \frac{\partial \eta}{\partial \alpha^e} d\alpha^e - \frac{\partial P}{\partial \gamma} d\gamma \quad (9) \]

Equation (9) can now be used to illustrate the relevance of aggregate demand price level elasticity to the determination of relative rates of inflation and hence the exchange rate.

First, and for convenience, suppose the two considered countries have zero growth rates of their nominal money supplies, \(dM = dM_f = 0\), and experience identical changes in price expectations, \(d\alpha^e > 0\), with stable aggregate demands, \(d\gamma_f = d\gamma = 0\). Substituting these values into equation (9) yields:

\[ d\pi = \left[ \frac{\partial P_f}{\partial \eta_f} \frac{\partial \eta_f}{\partial \alpha^e} - \frac{\partial P}{\partial \eta} \frac{\partial \eta}{\partial \alpha^e} \right] d\alpha^e \quad (10) \]

Then, substituting the solutions for \(\delta P/\delta \alpha^e\) and \(\delta P/\delta \alpha^e\) from (6f) and (6), respectively, results in:

\[ \frac{d\pi}{d\alpha^e} = \frac{P_f' \eta_f'}{c_2 - e_1} - \frac{P' \eta'}{a_2 - b_1} \quad (11) \]

where, \(P_f' \eta_f' = \frac{\partial P_f}{\partial \eta_f} \frac{\partial \eta_f}{\partial \alpha^e}\) and \(P' \eta' = \frac{\partial P}{\partial \eta} \frac{\partial \eta}{\partial \alpha^e}\).
The sign of equation (11) is easily determined with the following inequality conditions:

\[
\frac{d\pi}{d\alpha_e} > 0, \quad \text{if} \quad \left| \frac{c_2 + e_1}{P_f'\eta_f'} \right| > \left| \frac{a_2 + b_1}{P'\eta'} \right|
\]  \hspace{1cm} (12)

\[
\frac{d\pi}{d\alpha_c} > 0, \quad \text{if} \quad |a_\varepsilon| < |c_\varepsilon|
\]  \hspace{1cm} (12')

Equation (12) shows that the exchange rate varies positively with a negative (price-expectations-induced) aggregate supply shock when the weighted sum of the value of the foreign economy's aggregate price elasticities, \[ \left| \frac{c_2 + e_1}{P_f'\eta_f'} \right|, \] is greater than the comparable weighted sum for the home economy, \[ \left| \frac{a_2 + b_1}{P'\eta'} \right|. \] The weights are then the reciprocal products, \[ \frac{1}{P_f'\eta_f'} \text{ and } \frac{1}{P'\eta'} \], respectively, where each product reflects the magnitude of that nation’s aggregate supply response to a change in price expectations. Moreover, if we assume identical aggregate supply price level elasticities, \( b_1 = e_1 \) and equal weights, \( P_f'\eta_f' = P'\eta' \), it is clear that the nation with the less elastic aggregate demand function will experience appreciation of its currency when a negative (price-expectation’s induced) aggregate supply shock occurs. This condition is shown above in equation (12').

To further illustrate the relevance of aggregate demand price level elasticity for the price levels and exchange rate, again suppose the two considered countries have zero growth rates of their nominal money supplies, \( dM = dM_f = 0 \), and now identical positive aggregate demand.

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4 This conclusion also holds in the long-run when \( b_1 = e_1 = 0 \). When these parameters approach infinity, however, relative inflation and the equilibrium exchange rate are constant.
shocks, $d\gamma = d\gamma_f = d\overline{F} > 0$ with stable aggregate supply, $d\alpha^e = 0$. Substituting these into equation (9) yields:

$$d\pi = \left[ \frac{\partial P_f}{\partial \gamma_f} - \frac{\partial P}{\partial \gamma} \right] d\overline{F} \quad (13)$$

and substituting the solutions for $\delta P_f/\delta \gamma_f$ and $\delta P/\delta \gamma$ from (6f) and (6), respectively, leads to:

$$\frac{d\pi}{d\gamma} = -\frac{1}{c_2 - e_1} + \frac{1}{a_2 - b_1} \quad (14)$$

Signing (14) proceeds to the following inequality conditions:

$$\frac{d\pi}{d\gamma} > 0, \quad \text{if} \quad |a_2 - b_1| > |c_2 - e_1| \quad (15)$$

Equation (15) shows that the exchange rate varies positively with an aggregate demand shock

$$\frac{d\pi}{d\gamma} < 0, \quad \text{if} \quad |a_1| > |c_2| \quad (15')$$

when the sum of the absolute value of the foreign economy's aggregate elasticities is less than that same sum for the home economy. If we assume as above that $b_1 = e_1$, we obtain and it is clear that the nation with the more (less) elastic aggregate demand function will experience appreciation (depreciation) of its currency when aggregate demand shocks occur.\(^5\)

**Policy implications**

It is also noted that equations (15) and (15') can be similarly interpreted as demonstrating

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\(^5\) This conclusion also holds in the long-run when $b_1 = e_1 = 0$. When these parameters approach infinity, however, the equilibrium exchange rate is constant.
the likely impact of fiscal policy on the exchange rate. For instance and specifically, if one assumes expansionary fiscal policy by each nation, or $d\overline{\gamma} > 0$, this will result in currency depreciation in the nation having the less price-elastic aggregate demand function. This would then have subsequent balance of payments implications vis-à-vis the empirical J-Curve.

This model is also robust in that supply-side policies, even such as the central bank’s indirect control of expected inflation, as shown in equations (12) and (12’), can be shown in terms of the ultimate effect of this on the exchange rate and the balance of payments. In addition, the appendix details how the execution of like monetary policy money expansions by trading nations will lead to exchange rate movements so long as there are present differing aggregate-demand price elasticities.

Collectively, these policy results demonstrate the robustness of this model insofar as being able to extrapolate beyond the generalized exchange rate results to specific policy-oriented examples such as the fiscal and monetary policy examples described above.

**Summary and Conclusion**

This paper reaches two conclusions. First, with price-expectations or cost push related inflation, the effect on the exchange rate is ambiguous and depends, in part, on the price-level elasticity of aggregate demand. More specifically, when one nation has a less elastic aggregate demand relative to another, ceteris paribus, the exchange rate will rise (appreciation), while if aggregate demand is less elastic, the exchange rate will decrease (depreciation). Second, demand-pull inflation results unambiguously in an increase in the exchange rate and the magnitude of the appreciation of the home currency is greater the more elastic is the price-level elasticity of aggregate demand.
Also, if two economies have identical positive monetary growth rates and equivalent short-run aggregate supply conditions, the country having the higher aggregate demand elasticity with respect to the price level, will experience both a higher rate of inflation and currency depreciation.

Therefore, we find that the source of the inflation matters as well as aggregate demand elasticity for the direction of the exchange rate change. If two economies have unchanging nominal money supplies and experience equal aggregate demand or aggregate supply shocks which result in inflation, differences in the price-level elasticity of aggregate demand will determine the extent to which the exchange rate varies.
Appendix

Now, set the respective exogenous price expectations and supply shock variables \(d\alpha^e\), \(d\gamma\), and \(d\gamma^f\) equal to zero and assume that the countries' nominal money growth rates are positive and equal to an arbitrary constant, \(dM = dM^f = dM > 0\). These obtain:

\[
d\pi = \left[ \frac{\delta P_f}{\delta M_f} - \frac{\delta P}{\delta M} \right] dM \quad (1A)
\]

Substituting for \(\delta P_f/\delta M_f\) and \(\delta P/\delta M\) with derivatives obtained from equations (7f) and (7) and rearranging terms results in the following solution for \(d\pi/dM\):

\[
\frac{d\pi}{dM} = \frac{a_i}{a_2 - b_1} - \frac{c_i}{c_2 - e_1} > 0 \quad (2A)
\]

Equation (2A) reveals that the change in the exchange rate which results from equal nominal money supply increases in the two countries depends upon the price level elasticities of both aggregate demand and aggregate supply in the nations as well as the respective income velocities of money. For convenience, and to isolate the impact of aggregate demand elasticity on the exchange rate, let the income velocity of money equal one for both countries, i.e., \(a_1 = c_1 = 1\), and assume that the price level elasticities of aggregate supply in the two nations are also equal, \(b_1 = e_1 > 0\). These assumptions imputed to equation (2A) produce:

\[
\frac{d\pi}{dM} = \frac{1}{a_2 - b_1} - \frac{1}{c_2 - e_1} \geq 0, \quad \text{if} \quad c_2 - e_1 > a_2 - b_1 \quad (3A)
\]

\[
|a_2| > |c_2| \quad (3A)
\]

The interpretation of equation (12) is straightforward: for two economies with identical and unchanging short run aggregate supply conditions and equal growth rates of their respective nominal money supplies, that nation with the more elastic aggregate demand function with respect the price level will experience both a lower rate of inflation and a corresponding appreciation of its currency. In other words, the exchange rate would fall (rise) under the stated environment. From equation (3A) it follows that Cassel's posited relationship between relative rates of money growth and relative rates of inflation is valid only if the two nations have equal
aggregate demand elasticities.\textsuperscript{6}

\textsuperscript{6} In the long run, when $b_1=e_1=0$, equal growth rates of the nominal money supplies in the two countries will result in equal changes of the price levels and inflation rates and therefore no change in the equilibrium exchange rate. In the extreme Keynesian case, when $b_1=e_1\to\infty$, monetary growth would cause no change of the price level of either country and therefore no change of the equilibrium exchange rate.
References


